Group based problem

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Q:- Let $G = \{1, 5, 7, 11\}$ under multiplication modulo 12.

(a) Find the multiplication table of G.

(b) Find the order of each element

(c) Is G is cyclic?

Sol: We know $x \times_{12} y$ = remainder when the product xy is devided by 12

For example: $7 \times_{12} 11 = 5$

$5 \times_{12} 11 = 7$

$12)\overline{77}(6$
72
5

$12)\overline{55}(4$
48
7

$\times_{12}$	1	5	7	11
1	1	5	7	11
5	5	1	11	7
7	7	11	1	5
11	11	7	5	1

$12)\overline{25}(2$
24
1

Note:- Order of an element:- Let $a \in G$ be any arbitrary element, then order of an element is defined as the least positive integer, say n, such that $a^n = e$ where e is the identity of group G.

Then $O(a) = n$

<u>Order of an group</u> :- Number of elements in group G is called order of group G.

let $G = \{1, -1, i, -i\}$

$o(G) = 4$

(b). $1^1 = 1$ $\therefore o(1) = 1$

for order of 5, $5 \times_{12} 5 = 1$ $o(5) = 2$

For order of 7, $7 \times_{12} 7 = 1$ $o(7) = 2$

for order 11, $11 \times_{12} 11 = 1$ $o(11) = 2$

<u>Note</u> :- We know that a group G is cyclic, if there exists an element $a \in G$ such that $o(a) = o(G)$

for example; let $G = \{1, -1, i, -i\}$ $o(G) = 4$

$i^1 = i$ $= \{i^1, i^2, i^3, i^4\}$

$i^2 = -1$

$i^3 = -i$ $G = \langle i \rangle$

$i^4 = 1$

Here 1 is the identity element of group G.

$o(1) = 1$ $\{1^1 = 1$

$o(-1) = 2$ $\{(-1)^2 = 1$

$o(i) = 4$ $\{i^4 = 1$

$o(-i) = 4$ $\{(-i)^4 = 1$

Another way to show G is cyclic, when we prove that
$\exists\ a \in G$ s.t $o(a) = o(G)$

Here $\exists\ i \in G$ s.t $o(i) = o(G)$
that's why we say that G is cyclic

(c): $o(1) = 1$
$o(5) = 2$
$o(7) = 2$
$o(11) = 2$

$\therefore$ There is no element of G whose order equal to 4.

Hence G is not cyclic.